A Friend in Need....

Your friend is upset. She just got her exam on series back, and she lost points on every problem! She looked at the solutions, but still doesn't understand why her answers are wrong. Help your friend find her mistakes, and learn how to do the problems right before the final.

A) Find and circle an algebraic error or false statement in each problem.
B) Explain why what you circled is wrong. Come up with a counterexample if you can to help your friend.
C) Once you have found all the errors, complete the problems correctly on a separate sheet.

Every problem here IS wrong for some reason. If you can't find an error right away, move on to the next problem and come back later.

What do you think are the three 'worst' mistakes, that demonstate your friend really doesn't understand Chapter 10? Circle the numbers of these problems.

What should your friend do to get caught up?

1) $\int_0^\infty \frac{1}{e^x}\,dx$ converges since $\lim\limits_{n\to\infty} \dfrac{1}{e^x} = 0$.

2) $\int_0^\infty \frac{dx}{x^2}$ converges by the p-test.

3) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \sum_{n=1}^{\infty} \frac{1}{2n} - \sum_{n=1}^{\infty} \frac{1}{2n+4}$ Diverges since each sum diverges.

4) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n cos(\pi n)}{n}$
converges by the alternating series test since the terms alternate and go to zero.

5) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n}$ converges for $x = 1$ only, since this makes it a geometric series.

6) $$\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2} + \sum_{n=1}^{\infty} \frac{1}{3n^3}$$
Each of these series is a convergent p-series, so their sum converges.

7) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3} = \frac{1/3}{1-2} = -\frac{1}{3}$ by the formula for geometric series.

8) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n!}$ diverges by the ratio test since
$$\lim_{n\to\infty} \frac{(n+1)!}{n+1} \cdot \frac{n}{n!} = \lim_{n\to\infty} \frac{(n+1)(n)}{n+1} = \infty$$

9) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges since the sequence of partial sums $s_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ is always increasing, and so goes to infinity.

10) $\int_1^\infty \frac{1}{x^2-2x-1}\,dx$ converges by the comparison test since $\frac{1}{x^2-2x-1} \leq \frac{1}{x^2}$, and $\int_1^\infty \frac{1}{x^2}dx$ converges.